Midterm Introduction to Logic (CS&MA)

September 28th, 2014

☞ This midterm consists of five questions.

☞ Only write your student number at the top of the exam. Also put your number at the top of any additional pages.

☞ Put the name of your teaching assistant and group at the top of the exam.

☞ Use a blue or black pen (so no pencils, red pen or marker).

Good Luck!

1: translation into propositional logic (8 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

a. If Mitch works with the FBI, or does not leave the firm, he will lose his life as he knows it.

b. Only if he uncovers the identity of the one-armed man, is Kimble acquitted of first-degree murder.

c. If and only if Neveu deciphers the labyrinthine puzzle in time, the Priory's secret will not be lost forever.

2: translation into first-order logic (8 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key. Translate only names as constants.

a. If Ralph helps Vanellope, she helps him, unless Ralph gets a medal.

b. If Vanellope is chosen to race and Ralph does not stop her, she will die unless Ralph does not unplug the game.

c. Only if Vanellope crosses the finish line, she and Ralph both escape.

3: formal proofs (10 points) Give formal proofs of the following inferences.

a.

$$\neg(P \leftrightarrow \neg P)$$

b.
$$(P \to Q) \to Q$$
$$P \lor Q$$

c.
$$P \to Q$$
$$\neg P \to Q$$
$$Q$$

d.
$$a = b \to (a \neq c \to c \neq b)$$

4. Normal forms of propositional logic (7 points)

a. Provide a negation normal form (NNF) of: $\neg(P \land (\neg Q \lor \neg\neg R))$. Show all the intermediate steps.

b. Provide a disjunctive normal form (DNF) of: $\neg(P \land \neg Q) \land \neg(\neg S \lor R)$. Show all the intermediate steps.

5: Set theory (7 points)   Given the following three sets $A = \{1, \{2,3\}, \{5\}\}$, $B = \{\{2,3\}, 4, 5\}$ and $C = \{\{2,3\}, 5, \{5\}, 6\}$. For each of the following statements, determine whether it is true or false. You do not have to explain your answers.

a. $(A \cap B) \setminus C = \emptyset$

b. $3 \in A \cap B \cap C$

c. $(B \cap C) \setminus A \subseteq (A \cap C) \setminus A$

d. $6 \in C \cup \emptyset$

e. $A \cup B \subseteq B \cap C$

f. $\emptyset \setminus (A \cup B) \neq \emptyset$

g. $(A \setminus B) \setminus C = \{1\}$

h. $(A \cap B) \subseteq C$

i. $1 \notin (A \cup B) \cap (C \cup B)$

j. $C \setminus (C \cap B) \subseteq (A \cup B)$